LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Microeconomics

Masters in Economics and Masters in Monetary and Financial Economics

Solution Topics - Midterm Test

5th of November of 2015

Question 1

(4 marks) Show that if preferences ≿ are represented by a utility function, then ≿ satisfies completeness and reflexivity.

Consider any 2 consumption bundles x and y. Given that u(.) is a utility function and that ≥ (defined on the set of real numbers) is complete, we have u(x) ≥ u(y) or u(y) ≥ u(x). If the utility function u(.) represents ≿, by definition of utility function, we have x ≿ y or y ≿ x and ≿ is complete.

Consider a consumption bundle x. Given that u(.) is a utility function, we have u(x) ≥ u(x). If the utility function u(.) represents ≿, by definition of utility function, we have x ≿ x and ≿ is reflexive.

Question 2

A consumer has preferences over goods 1 and 2 represented by the utility function:

$$u(x_1, x_2) = \min\{2x_1, x_2\}.$$

Let p_1 be the price of good 1, let p_2 be the price of good 2, and let income be equal to y.

1. (3 marks) Derive the Marshallian demands for goods 1 and 2.

   At the optimum, $2x_1^* = x_2^*$ and $p_1x_1^* + p_2x_2^* = y$, then $x_1(y, p_1, p_2) = y/(p_1 + 2p_2)$ and $x_2(y, p_1, p_2) = 2y/(p_1 + 2p_2)$.

2. (1.5 marks) Derive the indirect utility function.

   $v(y, p_1, p_2) = 2y/(p_1 + 2p_2)$.

3. (1 mark) Use the Slutsky equation to decompose the effect of an own-price change on the demand for good 1 into income and substitution effects.

   Total effect is given by the derivative of $x_1(y, p_1, p_2)$ with respect to p_1, ie, $-y/(p_1 + 2p_2)^2$. Since the two goods are perfect complements, the substitution effect is zero and the income effect equals the total effect.

4. (1.5 marks) Determine the expenditure function.

   The expenditure function is the inverse of the indirect utility function. Thus, $e(u, p_1, p_2) = u(p_1 + 2p_2)/2$.

5. (2 marks) Show that the expenditure function is strictly increasing in *u*, increasing in prices, homogeneous of degree 1 in prices, and concave in prices.

The partial derivative of the expenditure function with respect to u is $(p_1 + 2p_2)/2$. Since $(p_1 + 2p_2)/2 > 0$ (for $p_1, p_2 > 0$), the expenditure function is strictly increasing in u.

The partial derivatives of the expenditure function with respect to p_1 and p_2 are u/2 and u, respectively. Since $u/2 \geq 0$ and $u \geq 0$ (for $u \geq 0$), the expenditure function is increasing in each price.

Since $e(u, tp_1, tp_2) = u(tp_1 + 2tp_2)/2 = tu(p_1 + 2p_2)/2 = te(u, p_1, p_2)$, for all $t > 0$, the expenditure function is homogeneous of degree one in prices.

Let $(p_1^t, p_2^t) = t(p_1^1, p_2^1) + (1-t)(p_1^2, p_2^2)$, $0 \leq t \leq 1$. Then is easy to show that $e(u, p_1^t, p_2^t) = te(u, p_1^1, p_2^1) + (1-t)\, e(u, p_1^2, p_2^2)$, so that $e(u, p_1^t, p_2^t) \leq te(u, p_1^1, p_2^1) + (1-t)\, e(u, p_1^2, p_2^2)$ and e(.) is concave in prices.

6. (1 mark) Using Shephard's lemma, derive the compensated (or Hicksian) demand functions.

Computing the partial derivative of the expenditure function with respect to each price, we obtain the Hicksian demand functions $x^h_1(y, p_1, p_2) = u/2$ and $x^h_2(y, p_1, p_2) = u$.

Question 3

(2 marks) Explain the Weak Axiom of Revealed Preference.

If a bundle of goods x is revealed preferred to x' (i.e., $p. x \geq p. x'$), then x' cannot be revealed preferred to x (i.e., $p' x > p' x'$).

Question 4

Consider the quadratic vNM-utility function $u(w) = a + bw + cw^2$, where w represents wealth.

1. (1 mark) What restrictions do the parameters a, b and c have to satisfy for this utility function to feature risk-aversion?

For $u'' < 0$, we must have $c < 0$. If $c < 0$, we must have $b > |2wc|$ for $u' > 0$. There are no restrictions on a.

2. (1 mark) For what range of w is the given function a reasonable utility function?

To have $u' > 0$, $w < b/(-2c)$.

3. (2 marks) Compute the coefficient of absolute risk-aversion and show that this function cannot exhibit diminishing absolute risk aversion if the restrictions in 1. are satisfied.

$R^a(w) = -u''/u' = -2c/(b + 2cw)$. Since the derivative of $R^a(w)$ with respect to w is positive (for any $b < |2wc|$ and $c < 0$), $R^a(w)$ cannot exhibit diminishing absolute risk aversion.